

Last year was a good year for the Lee's family business. During January 2012, revenues were \$18,371 and operating costs were \$5,841.

SCORE: ____ / 30 PTS

NOTE: You must show the use of the relevant sequence and/or series formulae to earn full credit for the following problems.

- [a] If revenues each month were 1.6% higher than revenues the previous month, what was the total revenue from January 2012 to August 2012 ?

$$\frac{18,371(1.016^8 - 1)}{1.016 - 1} = \$155,466.91$$

- [b] If operating costs each month decreased by a fixed amount (in dollars) from the previous month, and operating costs in August 2012 were \$4,742, what were the total operating costs from January 2012 to December 2012 ?

$$4742 = 5841 + 7d$$

$$d = -157$$

$$\frac{12}{2}(2(5841) + 11(-157)) = \$59,730$$

Consider the sequence defined recursively by $a_n = n^2 - 3a_{n-1}$, $a_1 = -1$.

SCORE: ____ / 10 PTS

Find the first 4 terms of the sequence, and write them as a list.

$$a_2 = 2^2 - 3a_1 = 4 + 3 = 7$$

$$a_3 = 3^2 - 3a_2 = 9 - 21 = -12$$

$$a_4 = 4^2 - 3a_3 = 16 + 36 = 52$$

-1, 7, -12, 52

Find rectangular equations corresponding to the parametric equations $x = \frac{t}{3-t}$, $y = \frac{t+2}{t-2}$.

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Write y as a function of x .

$$x(3-t) = t$$

$$3x - tx = t$$

$$3x = t + tx$$

$$3x = t(1+x)$$

$$t = \frac{3x}{1+x}$$

$$y = \frac{\frac{3x}{1+x} + 2}{\frac{3x}{1+x} - 2} \cdot \frac{1+x}{1+x}$$

$$y = \frac{3x + 2(1+x)}{3x - 2(1+x)}$$

$$y = \frac{5x+2}{x-2}$$

Prove by mathematical induction: $\sum_{i=1}^n 2i^2 = \frac{n(n+1)(2n+1)}{3}$ for all integers $n \geq 1$.

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HINT: The algebra in the proof is much easier if you use factoring.

$$\sum_{i=1}^1 2i^2 = 2(1)^2 = 2 = \frac{1(2)(3)}{3}$$

ASSUME $\sum_{i=1}^k 2i^2 = \frac{k(k+1)(2k+1)}{3}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$$\begin{aligned}\sum_{i=1}^{k+1} 2i^2 &= \sum_{i=1}^k 2i^2 + 2(k+1)^2 \\ &= \frac{k(k+1)(2k+1)}{3} + 2(k+1)^2 \\ &= (k+1) \left[\frac{k(2k+1)}{3} + 2(k+1) \right]\end{aligned}$$

$$= (k+1) \left[\frac{2k^2 + k + 6k + 6}{3} \right]$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{3}$$

$$= \frac{(k+1)(k+2)(2k+3)}{3}$$

SO, BY MATHEMATICAL INDUCTION,

$$\sum_{i=1}^n 2i^2 = \frac{n(n+1)(2n+1)}{3}$$

FOR ALL INTEGERS
 $n \geq 1$

Find the coefficient of $x^{16}y^{42}$ in the expansion of $(9x^4 - 11y^2)^{25}$.

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Your final answer may use $+$, $-$, $\times$ and positive powers. It may **NOT** use $\dots$, $!$, $\div$, negative exponents **NOR** fractions.

It does **NOT** need to be simplified into a single number. Show all work as demonstrated in lecture.

$$\begin{aligned} & \sum_{i=0}^{25} \binom{25}{i} (9x^4)^{25-i} (-11y^2)^i \\ &= \sum_{i=0}^{25} \binom{25}{i} 9^{25-i} x^{4(25-i)} (-11)^i y^{2i} \\ & \quad 4(25-i)=16 \text{ AND } 2i=42 \\ & \quad \text{IF } i=21 \end{aligned}$$

$$\begin{aligned} & \binom{25}{21} 9^4 (-11)^{21} \\ &= \frac{25 \cdot 24 \cdot 23 \cdot 22}{4 \cdot 3 \cdot 2 \cdot 1} \cdot 9^4 (-11)^{21} \\ &= 25 \cdot 23 \cdot 22 \cdot 9^4 \cdot (-11)^{21} \end{aligned}$$

Use sigma notation to write the series $\frac{7}{75} - \frac{14}{71} + \frac{28}{67} - \dots + \frac{1792}{43}$. $\leftarrow$ GEOMETRIC $r = -2$
 $\leftarrow$ ARITHMETIC $d = -4$

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Show clearly the algebra used to find the upper limit of summation (ie. not just by counting).

$$\sum_{n=1}^9 \frac{7 \cdot (-2)^{n-1}}{75 - 4(n-1)} = \sum_{n=1}^9 \frac{7 \cdot (-2)^{n-1}}{79 - 4n}$$

$$79 - 4n = 43$$

$$\text{IF } n = 9$$

Find parametric equations for the ellipse with $(-1, 7)$ and $(-1, -5)$ as foci, and $(3, 1)$ as one endpoint of the minor axis.

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$$F \cdot (-1, 7)$$

$$C \cdot (-1, 1) \cdot (3, 1)$$

$$F \cdot (-1, -5)$$

$$a^2 = 6^2 + 4^2$$

$$a^2 = 52$$

$$a = 2\sqrt{13}$$

$$x = -1 + 4 \cos t$$

$$y = 1 + 2\sqrt{13} \sin t$$